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Speed of a surge in a bathtub vortex

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The surge studied in this paper is due to the sudden closure of the sink of a bathtub vortex. Experimental evidence established that the surge is not initially a wave but is in fact a slug of fluid which has, for a very short period of time, a distinctive motion of its own. The purpose of this paper is to present the salient features of an experimental investigation of the effect of circulation, geometry, and viscosity on the speed of the surge and to report some rather unexpected observations.

Experimental apparatus

The experimental investigation was performed in a cylindrical vortex tank, illustrated schematically in figure 1, using water as the working fluid. The apparatus is similar to that described by Granger (1966).

The tank was 8 ft. high and 4 ft. in diameter and could be filled with water to within an inch of its rim. It was constructed out of two concentric cylindrical walls. The outer cylinder, made of steel with numerous plastic windows in its surface, was firmly mounted in a horizontal tank table so as to create a water tight container. An inner cylinderical wall made of clear plastic was inserted into the outer cylinder to form an annular space of $\frac{1}{2}$ in. radial width. The water circulating through the tank was admitted to this annular space through vertical tubes equally spaced about the perimeter, the walls of the tubes perforated by equally spaced small holes so as to distribute the entering water uniformly in a vertical direction. Numerous jet orifices were punched with uniform spacing in the inner cylindrical wall to direct the jets of water tangentially along the inner cylindrical surface. In this way, a circulation was imparted to the body of water filling the central space of the tank. The magnitude and distribution of circulation throughout the field of laminar flow of the vortex were measured by timing the motion of a globule of dye judiciously inserted into the water so as to impart an initial momentum equal to that of the ambient fluid. The path of the globule, in general helical, appeared nearly circular when viewed parallel to the axis of the tank. The radius of such a path was measured and the angular velocity of the globule was clocked over a complete circle, or over a marked arc. Care was taken to obviate errors in measurement ensuing from parallax effects. Also, precautions were taken to preclude possible errors due to precessional motion of the vortex filament by eliminating measurements for which such precession occurred. The circulated water was removed through a central hole located on the bottom surface of the tank. Thence the water passed through a remotecontrolled shut-off valve to a centrifugal pump placed into the external closed circuit, then through a throttle valve and finally to a header back into the admission tubes of the tank.

The non-steady perturbations, whether transient or oscillatory, were introduced either at the sink point, that is, the bottom outlet of the vortex tank, or at the point of intersection of the axis of the vortex with the free surface. In the former case, variations of efflux of controlled magnitude and duration were produced by a shut-off valve. This mechanism consisted of a rotating valve operable

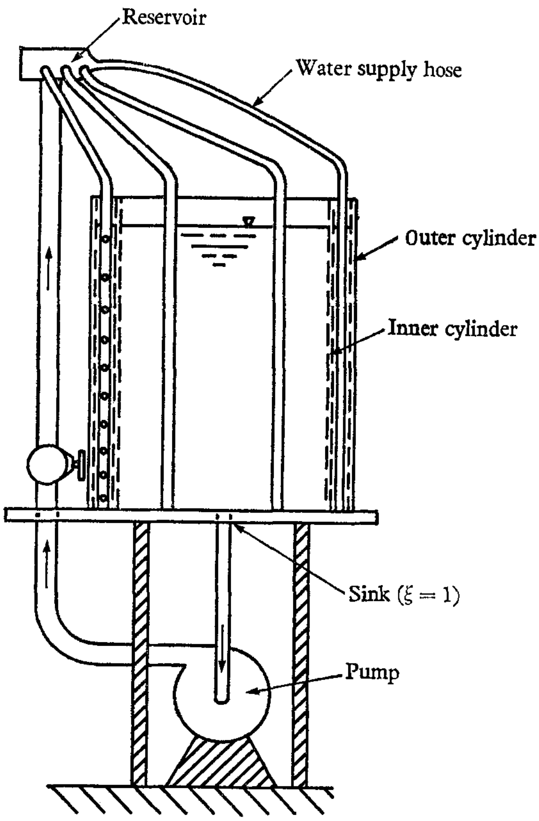

FIGURE 1. Schematic diagram of vortex generator.

at forcing frequencies β from 0 to 780 rev/min, producing two cycles of operation per revolution, and a by-pass valve arranged in such a manner that a decrease of rate of efflux ranging from 0 to 100 % of total sink strength could be effected by closure of the rotating valve.

Perturbations could be introduced at the free surface by inserting and retracting a circular disk lying in the plane of the water surface with variation of frequency and intensity corresponding to those at the sink point. For oscillatory excitation, the valve was operated on 50 % open, 50 % closed cycle. At zero frequency, the signal was reduced to a step function, or else, after closure of the valve, the valve was re-opened after a predetermined time interval.

Formation of the pulse

The development of the surge was studied with the use of 35 mm film at the rate of 300 frames per second. Paraphrasing Lambourne's (1965) description, the development of the surge is as follows:

When the sink is closed, the narrow filament of dye at the centreline thickens near the sink. Use of a Pitot tube verified the theoretical prediction that the pressure near the sink at the centreline has suddenly increased and the fluid in this region must move outward radially. The thickening rapidly develops into a surface very much like that of a bulb. The fluid in the interior has decelerated to a near-stagnant state. Fluid still flows over the bulbous shape. The sudden creation of a stagnant condition at the sink provides conditions for the bulbous shape to move towards the free surface.

Owing to the upward motion of the bulbous shape, a re-entrant jet is soon formed around the bottom surface such that fluid now flows upstream and within the object. With a near-stagnant state existing in the interior, a vortex ring is created and the bulbous shape closes upon itself, forming a sphere with a vortex ring inside. Owing to pressure instabilities in the wake the vortex ring soon breaks apart and an emptying process occurs in which the trapped fluid is shed downstream along the vortex filament as shown in figure 2 (plate 1). Note that, in the figure, the position of the vortex ring is inclined inside the bulb and fluid is being shed from the upper ring.

With the circulatory strength of the vortex ring diminished, figure 3 (plate 2) shows the bulbous shape transformed to a single spiral filament. The filament will remain approximately in this position until its obliteration at the free surface.

All the above stages occur in the formation and breakdown of the pulse while the sink remains closed. By careful manipulation of the shut-off valve so that the sink is momentarily closed, one can maintain the vortex ring inside the bulbous shape for a considerable period of time. This is attributed to the negligible magnitude of the pressure instabilities in the wake; this results from the restoration of pre-closure conditions at the sink (see figure 4, plate 3).

In response to step function excitation, the initial pulse is sometimes followed by a second, though considerably weaker, pulse, and, under certain conditions, by a third pulse, which is barely discernible. The occurrence of periodic pulses of larger than ordinary amplitudes for a certain range of excitation frequencies was taken to be evidence of the existence of a resonance condition. Corresponding characteristics were observed when inducing disturbances at the free surface. Downward absolute speeds were greater than upward absolute speeds, as were the speeds of downward pulses created by reflexion of upward pulses at the free surface. Figure 6 shows that the relative speed was found to be essentially constant for instantaneous and for periodic excitation, no matter what the frequency of the latter.

Speed of the disturbance

Measurements were made of the speed of propagation of the pulse, at several dimensionless positions ξ along the axis of the vortex and for different values of the circulation of the vortex field. Series of measurements were taken at each location and for each circulation, each at a different frequency

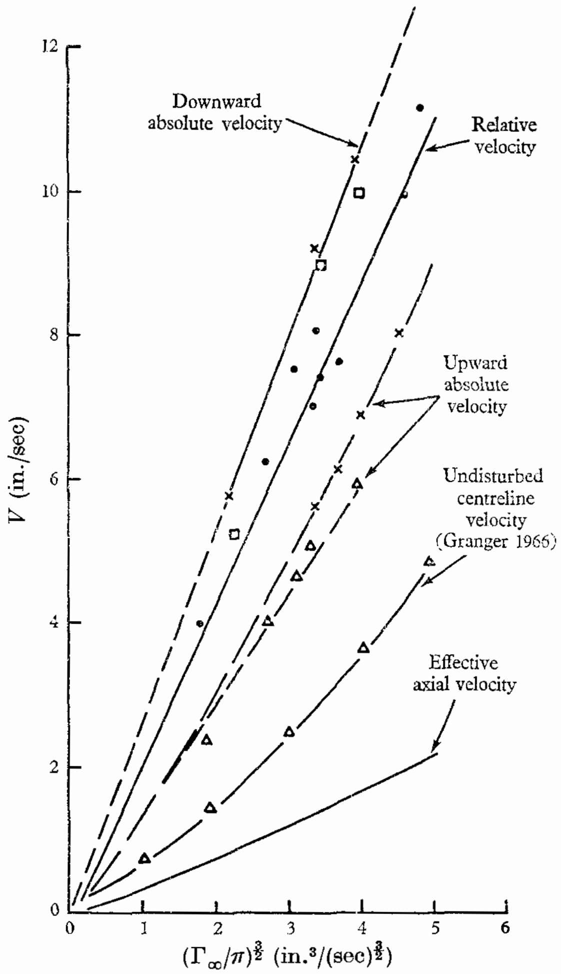

FIGURE 5. Axial velocity at the centreline *versus* circulation. $\triangle$, $\xi = 0{\cdot}62$; $\times$, $\xi = 0{\cdot}38$; $\square$, $\xi = 0{\cdot}13$.

of excitation. The results obtained cover the range of frequencies for which response was observed. Measurements were obtained also at zero frequency (step function excitation). The absolute velocity of surge propagation was measured by optical timing of the interval of travel between referenced markers placed along the axial extent of the vortex tube.

As previously stated, the velocities of propagation of the pulse along the axis of the vortex, when measured in co-ordinates fixed with respect to the tank, were much larger for downward than for upward travel of the surge. A limited number of absolute speeds of propagation were obtained for both upward and downward propagation at specific locations along the vortex axis covering the same range of circulations. These are plotted in figure 5.

The effect of viscosity on the surge velocity was determined by varying the temperature of the working fluid. Water temperature of 72, 108 and 130 °F produced sufficient changes in kinematic viscosity to verify that the surge velocity was inversely proportional to the square root of the kinematic viscosity. Introducing various size sink orifices produced the result that the surge velocity was inversely proportional to the diameter of the sink. Generation of step function excitation was unsatisfactory for sink diameters greater than 1 in.

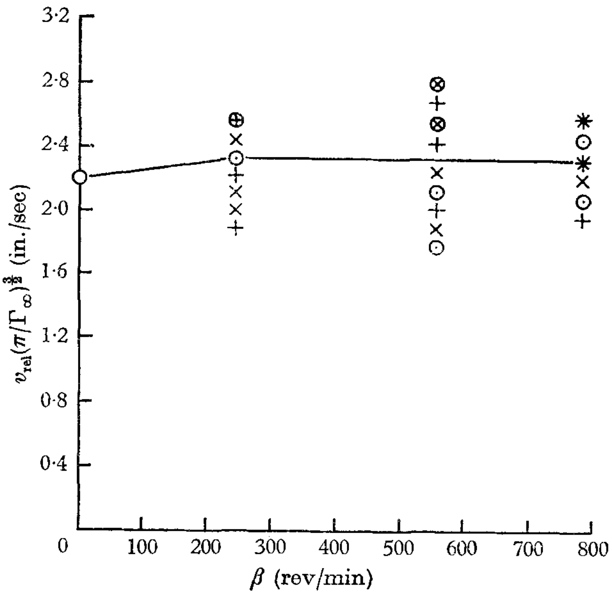

FIGURE 6. Variation of forcing frequency with relative surge velocity. $\times$, $\xi = 0{\cdot}32$; $\otimes$, $\xi = 0{\cdot}44$; $\odot$, $\xi = 0{\cdot}58$; $+$, $\xi = 0{\cdot}86$.

The effective axial velocity in the core was then compared with the centreline axial velocity given in Granger (1966, figure 8) for the undisturbed state at the same axial location. The effective velocity is defined as the mean value of the axial velocity at the centreline and the axial velocity based on a Gaussian distribution evaluated at the core radius.

It is seen in figure 5 that the effective axial velocity in the core varies similarly to the variational centreline axial velocity. The effective axial velocity was measured at one axial station only. Proceeding on the assumption that deviations of the ratio of the undisturbed state axial velocity to effective axial velocity at other axial locations from the stated value are small, the determination of the relative velocity of propagation was made in the following manner. A number of measurements of upward absolute velocity of propagation were made at several axial positions for which corresponding downward velocities were not available. The derived effective axial velocity was then added to the corresponding measured upward absolute velocity. The resulting points of the relative velocity, shown as dots in figure 5, do not disclose a systematic variation with axial location; conversely, they furnish evidence that the relative velocity appears to be constant along the extent of the vortex core, since the mean values fall along a straight line. (The spread of the dots reflects the experimental error of measurement.) These results also indicate that the relative velocity varies proportionally to (circulation)$^{\frac{3}{2}}$. Measurements of absolute upward surge velocity, too, were attempted close to the sink. These results, however, proved too inaccurate to furnish precise results, because of unavoidable errors in timing. They were found to confirm the conclusion stated above.

Hence, it was observed that: (i) the surge generated either at the sink point ($\xi = 1$) or at the intersection of the vortex axis with the free surface of the water advanced along the vortex axis at a constant speed with respect to the fluid for a given value of circulation. This result is assumed to apply to disturbances proceeding toward the sink after reflexion at the free surface; (ii) the relative speed was found to be essentially constant for instantaneous and for periodic excitation, regardless of the frequency of the latter; (iii) the relative surge velocity varied as the three-halves power of the circulation, inversely as the sink diameter and inversely as the square root of the kinematic viscosity; (iv) the disturbance is not initially a wave, but is a slug of fluid with an orderly motion independent of the motion of the external field.

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FIGURE 2. Inclined vortex ring inside surge.

FIGURE 3. Origin of the spiral filament.

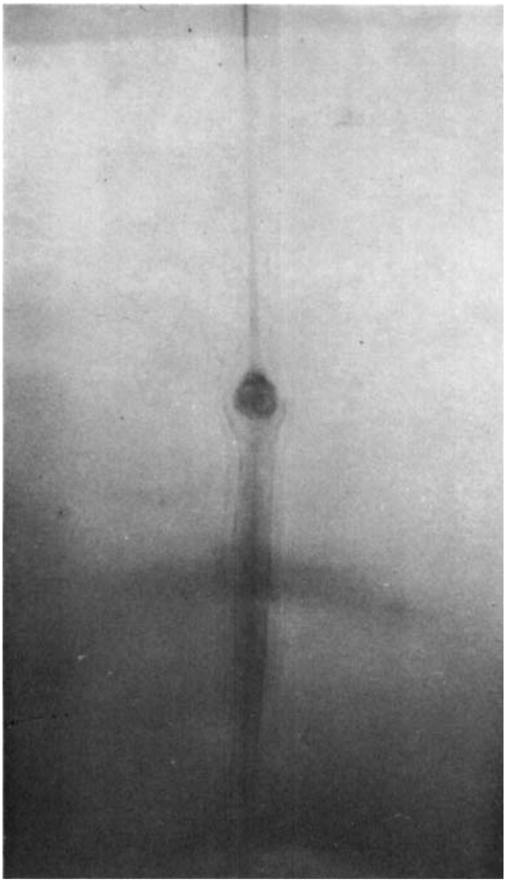

FIGURE 4. A spherical surge.